

Find the simplex edge length x .

Question:

What is the value of edge length x in all n dimensions, where the volume v of the regular simplex equals one?

Answer:

The volume V of a regular simplex is:

$$V = \frac{x^n}{n!}$$

Where n is the dimension and x is the length of the edges.

$$\frac{x^n}{n!} = 1 \Rightarrow x^n = n!$$

$$\Rightarrow n \ln x = \ln(n!)$$

$$\ln x = \frac{\ln(n!)}{n}$$

$$x = e^{\frac{\ln(n!)}{n}}$$

Check: in the case where $n = 3$.

$$x = e^{\frac{\ln(3!)}{3}}$$

$$\begin{aligned} &= e^{\frac{\ln(6)}{3}} \\ &= 1.817120593 \text{ to } 9 \text{ d.p.} \end{aligned}$$

We know that a regular tetrahedron has volume

$$V = \frac{x^3}{6} = 1$$

where $V = 1$

$$x^3 = 6$$

$$x = 6^{\frac{1}{3}}$$

$$= 1.817120593 \text{ to } 9 \text{ d.p.}$$

$$\textit{therefore } e^{\frac{\ln(n!)}{n}} \equiv (n!)^{\frac{1}{n}}$$