

This is the M500 Society Problem 192.6 - 500 Factors

Find the smallest number with 500 factors?

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Solution 192.6 - 500 Factors:

First I looked at a smaller number

48 has 10 factors. This can be worked out by looking at the prime numbers involved

$$2^4 \cdot 3^1 = 48$$

I discovered that the number of factors is equal to the power of indices of prime numbers plus one.

$$2^4 \cdot 3^1 \text{ becomes } (4+1)(1+1) = 5 \cdot 2$$

So the smallest number to have 500 factors is made up from:

$$5 \cdot 5 \cdot 5 \cdot 2 \cdot 2 \text{ prime factors} = 500 \text{ factors.}$$

Working backwards, minus one from the above prime factors to get:

$$5-1, 5-1, 5-1, 2-1, 2-1, = 4, 4, 4, 1, 1,$$

Put them into the powers of prime numbers:

$$2^4 \cdot 3^4 \cdot 5^4 \cdot 7^1 \cdot 11^1$$

And we get the answer 62370000.

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Checking the answer with the html file below:

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```
<!DOCTYPE HTML PUBLIC " - //W3C//DTD XHTML 1.0 Transitional//EN"
"http://www.w3.org/TR/xhtml1/DTD/xhtml1-transitional.dtd"
">
<html xmlns="http://www.w3.org/1999/xhtml">
<head>
<title>Adi Cox Mathematics</title>
</head>
<body>
<script type="text/javascript" language="javascript">
```

```
document.write("<h3>The square root of x</h3>")
```

```
u=1;
```

```
w=62370000;
```

```
n=2;
```

```
x=1;
```

```

v=100;

for (b = u; b <= v; b++)
{

x=((1/2)*(x+(w/x)))

}

document.write(w,"^",1/n," = ", x,"<br />");

```

```

document.write("<h3>The factors of x<h3>")
v=x;

for (b = u; b <= v; b++)
{

if(Math.round(w/b)==w/b)
    {
    document.write(w,"/",b," = ", w/b,"<br />");
    }

}

```

```

</script>
</body>
</html>

```

---

The output we get is of the the square root of 62370000 followed by the 500 factors:

```

62370000/1 = 62370000
62370000/2 = 31185000
62370000/3 = 20790000
62370000/4 = 15592500
62370000/5 = 12474000
62370000/6 = 10395000
62370000/7 = 8910000
62370000/8 = 7796250
62370000/9 = 6930000
62370000/10 = 6237000
62370000/11 = 5670000
62370000/12 = 5197500
62370000/14 = 4455000
62370000/15 = 4158000
62370000/16 = 3898125
62370000/18 = 3465000
62370000/20 = 3118500
62370000/21 = 2970000

```

62370000/22 = 2835000  
62370000/24 = 2598750  
62370000/25 = 2494800  
62370000/27 = 2310000  
62370000/28 = 2227500  
62370000/30 = 2079000  
62370000/33 = 1890000  
62370000/35 = 1782000  
62370000/36 = 1732500  
62370000/40 = 1559250  
62370000/42 = 1485000  
62370000/44 = 1417500  
62370000/45 = 1386000  
62370000/48 = 1299375  
62370000/50 = 1247400  
62370000/54 = 1155000  
62370000/55 = 1134000  
62370000/56 = 1113750  
62370000/60 = 1039500  
62370000/63 = 990000  
62370000/66 = 945000  
62370000/70 = 891000  
62370000/72 = 866250  
62370000/75 = 831600  
62370000/77 = 810000  
62370000/80 = 779625  
62370000/81 = 770000  
62370000/84 = 742500  
62370000/88 = 708750  
62370000/90 = 693000  
62370000/99 = 630000  
62370000/100 = 623700  
62370000/105 = 594000  
62370000/108 = 577500  
62370000/110 = 567000  
62370000/112 = 556875  
62370000/120 = 519750  
62370000/125 = 498960  
62370000/126 = 495000  
62370000/132 = 472500  
62370000/135 = 462000  
62370000/140 = 445500  
62370000/144 = 433125  
62370000/150 = 415800  
62370000/154 = 405000  
62370000/162 = 385000  
62370000/165 = 378000  
62370000/168 = 371250  
62370000/175 = 356400  
62370000/176 = 354375  
62370000/180 = 346500  
62370000/189 = 330000  
62370000/198 = 315000  
62370000/200 = 311850  
62370000/210 = 297000  
62370000/216 = 288750  
62370000/220 = 283500  
62370000/225 = 277200  
62370000/231 = 270000  
62370000/240 = 259875

62370000/250 = 249480  
62370000/252 = 247500  
62370000/264 = 236250  
62370000/270 = 231000  
62370000/275 = 226800  
62370000/280 = 222750  
62370000/297 = 210000  
62370000/300 = 207900  
62370000/308 = 202500  
62370000/315 = 198000  
62370000/324 = 192500  
62370000/330 = 189000  
62370000/336 = 185625  
62370000/350 = 178200  
62370000/360 = 173250  
62370000/375 = 166320  
62370000/378 = 165000  
62370000/385 = 162000  
62370000/396 = 157500  
62370000/400 = 155925  
62370000/405 = 154000  
62370000/420 = 148500  
62370000/432 = 144375  
62370000/440 = 141750  
62370000/450 = 138600  
62370000/462 = 135000  
62370000/495 = 126000  
62370000/500 = 124740  
62370000/504 = 123750  
62370000/525 = 118800  
62370000/528 = 118125  
62370000/540 = 115500  
62370000/550 = 113400  
62370000/560 = 111375  
62370000/567 = 110000  
62370000/594 = 105000  
62370000/600 = 103950  
62370000/616 = 101250  
62370000/625 = 99792  
62370000/630 = 99000  
62370000/648 = 96250  
62370000/660 = 94500  
62370000/675 = 92400  
62370000/693 = 90000  
62370000/700 = 89100  
62370000/720 = 86625  
62370000/750 = 83160  
62370000/756 = 82500  
62370000/770 = 81000  
62370000/792 = 78750  
62370000/810 = 77000  
62370000/825 = 75600  
62370000/840 = 74250  
62370000/875 = 71280  
62370000/880 = 70875  
62370000/891 = 70000  
62370000/900 = 69300  
62370000/924 = 67500  
62370000/945 = 66000  
62370000/990 = 63000

62370000/1000 = 62370  
62370000/1008 = 61875  
62370000/1050 = 59400  
62370000/1080 = 57750  
62370000/1100 = 56700  
62370000/1125 = 55440  
62370000/1134 = 55000  
62370000/1155 = 54000  
62370000/1188 = 52500  
62370000/1200 = 51975  
62370000/1232 = 50625  
62370000/1250 = 49896  
62370000/1260 = 49500  
62370000/1296 = 48125  
62370000/1320 = 47250  
62370000/1350 = 46200  
62370000/1375 = 45360  
62370000/1386 = 45000  
62370000/1400 = 44550  
62370000/1485 = 42000  
62370000/1500 = 41580  
62370000/1512 = 41250  
62370000/1540 = 40500  
62370000/1575 = 39600  
62370000/1584 = 39375  
62370000/1620 = 38500  
62370000/1650 = 37800  
62370000/1680 = 37125  
62370000/1750 = 35640  
62370000/1782 = 35000  
62370000/1800 = 34650  
62370000/1848 = 33750  
62370000/1875 = 33264  
62370000/1890 = 33000  
62370000/1925 = 32400  
62370000/1980 = 31500  
62370000/2000 = 31185  
62370000/2025 = 30800  
62370000/2079 = 30000  
62370000/2100 = 29700  
62370000/2160 = 28875  
62370000/2200 = 28350  
62370000/2250 = 27720  
62370000/2268 = 27500  
62370000/2310 = 27000  
62370000/2376 = 26250  
62370000/2475 = 25200  
62370000/2500 = 24948  
62370000/2520 = 24750  
62370000/2625 = 23760  
62370000/2640 = 23625  
62370000/2700 = 23100  
62370000/2750 = 22680  
62370000/2772 = 22500  
62370000/2800 = 22275  
62370000/2835 = 22000  
62370000/2970 = 21000  
62370000/3000 = 20790  
62370000/3024 = 20625  
62370000/3080 = 20250

62370000/3150 = 19800  
62370000/3240 = 19250  
62370000/3300 = 18900  
62370000/3375 = 18480  
62370000/3465 = 18000  
62370000/3500 = 17820  
62370000/3564 = 17500  
62370000/3600 = 17325  
62370000/3696 = 16875  
62370000/3750 = 16632  
62370000/3780 = 16500  
62370000/3850 = 16200  
62370000/3960 = 15750  
62370000/4050 = 15400  
62370000/4125 = 15120  
62370000/4158 = 15000  
62370000/4200 = 14850  
62370000/4375 = 14256  
62370000/4400 = 14175  
62370000/4455 = 14000  
62370000/4500 = 13860  
62370000/4536 = 13750  
62370000/4620 = 13500  
62370000/4725 = 13200  
62370000/4752 = 13125  
62370000/4950 = 12600  
62370000/5000 = 12474  
62370000/5040 = 12375  
62370000/5250 = 11880  
62370000/5400 = 11550  
62370000/5500 = 11340  
62370000/5544 = 11250  
62370000/5625 = 11088  
62370000/5670 = 11000  
62370000/5775 = 10800  
62370000/5940 = 10500  
62370000/6000 = 10395  
62370000/6160 = 10125  
62370000/6237 = 10000  
62370000/6300 = 9900  
62370000/6480 = 9625  
62370000/6600 = 9450  
62370000/6750 = 9240  
62370000/6875 = 9072  
62370000/6930 = 9000  
62370000/7000 = 8910  
62370000/7128 = 8750  
62370000/7425 = 8400  
62370000/7500 = 8316  
62370000/7560 = 8250  
62370000/7700 = 8100  
62370000/7875 = 7920